

MEMORANDUM

To: Stan Carpenter

From: William Keyes

Date: 13 September 1965

Subject: HYPERGEOMETRIC DISTRIBUTION:1. EXPONENTIAL FAILURE LAW:

To approximate the actual failure distribution, the exponential failure law is assumed. Let F denote the number of components which would fail when N components of $MTTF = M$ hours are tested for t hours each. Then

$$F = N \left(1 - e^{-\frac{t}{M}} \right) \dots \dots \dots \quad (1)$$

11. HYPERGEOMETRIC DISTRIBUTION:

Given a lot of N gyros of which F (determined from equa. (1)) are defective, what is the probability of finding i defectives in a random sample of n units?

Let X = hypergeometric random variable, the number of defectives drawn from a sample of n gyros of which F are defective.

$$\begin{aligned} P(X = i) &= \frac{\text{No. of favorable events}}{\text{No. of possible events}} \\ &= \frac{\text{No. of ways to get } i \text{ defectives and } (n-i) \text{ non-defectives}}{\text{No. of ways to select } n \text{ samples from lot size } N} \end{aligned}$$

$$\begin{aligned} \text{Numerator} &= (\# \text{ ways to select } i \text{ defectives from } F \text{ defectives}) \\ &\quad \times (\# \text{ ways to select } (n-i) \text{ non-defectives from original } (N-F) \text{ lot of non-defectives}) \\ &= \binom{F}{i} \binom{N-F}{n-i} \quad \text{where } \binom{F}{i} = \frac{F!}{i! (F-i)!} \quad \text{etc.} \end{aligned}$$

Denominator = # Combinations of N objects taken n at a time.

$$= \binom{N}{n}$$

Let $p(i; N, F, n)$ = probability of finding i defectives in a sample of n gyros drawn at random from a lot of N gyros of which F are defective.

Then $p(i; N, F, n) = \frac{\binom{F}{i} \binom{N-F}{n-i}}{\binom{N}{n}}$ (2)

The sum of hypergeometric probabilities must be 1

$$\sum_{i=0}^N \frac{\binom{F}{i} \binom{N-F}{n-i}}{\binom{N}{n}} = 1$$

defining $\binom{N}{n} = 0$ when $n > N$

With an acceptance number of failures C, a partial sum determines confidence factor:

$$\sum_{i=0}^C p(i; N, F, n) = \text{sum of probabilities of finding } i = 0, 1, \dots, C \text{ defectives}$$

Define confidence factor P

$$\sum_{i=0}^C p(i; N, F, n) = \sum_{i=0}^C \frac{\binom{F}{i} \binom{N-F}{n-i}}{\binom{N}{n}} \equiv 1 - P \dots (3)$$

To establish an acceptance test which will project a MTTF of $M = \sqrt{rt}$ (t = testing time) with confidence P from a lot of N units with acceptance number C , the number of samples, n , which are needed is found:

- a. Calculate F from eq. (1) to nearest integer
- b. Calculate n from eq. (3)

If n units from a lot of N gyros are tested for t hours each and C random failures are observed, the MTTF with confidence P is calculated:

- a. Solve equa.(3) for F
- b. Solve equa. (1) for M

EXAMPLE:

In a procurement of 100 units, the MTTF is required to be 100,000 hours with 80% confidence and acceptance no. of failures $C = 1$. Find minimum sample size for 10,000 hours of testing.

$$\text{a. } F = N \left(1 - e^{-\frac{t}{M}} \right) = 100 \left(1 - e^{-0.1} \right) = 10$$

$$\text{b. } \frac{\binom{F}{0} \left(\binom{N-F}{n} + \binom{F}{1} \binom{N-F}{n-1} \right)}{\binom{N}{n}} \leq 1 - P$$

try $n = 20$

$$\frac{\binom{90}{20} + 10 \binom{90}{19}}{\binom{100}{20}} = .42 \text{ which is } > \text{ than the acceptable "risk" of } 1-P=.20$$

∴ 20 samples is too few

try $n = 30$

$$\frac{\binom{90}{30} + 10 \times \binom{90}{29}}{\binom{100}{30}} = .194 \text{ which is within the acceptable "risk" of .20}$$

Therefore if 30 units are tested for 10,000 hours each and 1 random failure is observed, the MTTF of each of the original lot of 100 units is 100,000 hours with 80% confidence.

W. Keyes

William Keyes

To approximate the failure rate of the lot, the random failure rate is assumed. Let k be the number of random failures from a sample of size n in a component of MTTF testing. The expected failure rate is then

12. WHAT SOMETHING TO YOU SAY?

Given a lot of N units of a given component, with a 1% failure rate, what is the probability of having k or more failures in a sample of n units?

Let X be a random variable representing the number of failures in a sample of n units. Then the distribution of X is given by

$$P(X = k) = \frac{N!}{k!(N-k)!} \left(\frac{1}{N}\right)^k \left(\frac{N-1}{N}\right)^{N-k}$$

For example, if $N = 100$ and $n = 10$, then $P(X = 1) = 0.272$ and $P(X = 2) = 0.226$.

It is of interest to note that the probability of observing 1 or 2 failures in a sample of 10 units is approximately 0.500. This is because

$$P(X = 0) + P(X = 1) + P(X = 2) = \frac{100!}{0!99!} \left(\frac{1}{100}\right)^0 \left(\frac{99}{100}\right)^{100} + \frac{100!}{1!98!} \left(\frac{1}{100}\right)^1 \left(\frac{99}{100}\right)^{99} + \frac{100!}{2!98!} \left(\frac{1}{100}\right)^2 \left(\frac{99}{100}\right)^{98} \approx 0.500$$